Time: 2 Hrs.30 Mins. Marks: 75

N.B. (i) All questions are compulsory.

(ii) Attempt ANY TWO-sub questions out of FOUR in Q1, Q2 and Q3.

(iii) Attempt ANY THREE-sub questions from Q4.

(ii) Figures to the right indicate marks.

Q.1. (a) Define for a continuous random variable, its 10

(i) probability density function

(ii) cumulative distribution function.

State two properties of each of them.

(b) For a continuous random variable, p.d.f. is 10

$f(x) = kx^2 (1-x) \quad 0 < x < 1$

$= 0 \quad$ otherwise

Find k, mean and standard deviation of X.

(c) A random variable X has p.d.f. as 10

$f(x) = kx \quad 0 \le x < 1$

$= 2k \quad 1 \le x < 2$

$= kx - 2k \quad 2 \le x < 3$

$= 0 \quad$ otherwise

Find k. Hence determine the cumulative distribution function F(x). Draw a sketch of f(x) and F(x).

(d) The life in hours (X) of a certain kind of tubes has the probability density function 10

$f(x) = \frac{100}{x2} \quad x \ge 100$

$= 0 \quad$ otherwise

Determine the form of cumulative distribution function of X. Hence obtain the probability that the tube would last up to 150 hours.

Q.2. (a) State the probability density function of an exponential variate X with mean θ. Also obtain expressions for mean and variance of X. 10

(b) The distribution of number of words written per day by a certain writer over a period of one year showed Rectangular distribution over (1000, 2000). Find the chance that on a randomly chosen day of the year he wrote 10

(i) at least 1200 words

(ii) Anywhere from 1250 to 1750 words.

(c) The bulbs used by the Municipality for general lighting have an average life of 500 burning hours with a standard deviation of 100 hours. How many of 10000 bulbs will have to be replaced 10

(i) within first 400 hours

(ii) between 250 and 450 hours?

(Assume that the life of the bulbs follow normal distribution.)

(d) Find the mean and standard deviation of a normal distribution of marks in an examination where 50% of the candidates obtained marks below 75 ; 4% got above 80 and the rest between 75 and 80.(For a SNV t, the area under the curve between $t = \pm 2$ is 0.96 and between $t = \pm 1.8$ is 0.92) 10

PQ3AAH

Q.3. (a) Explain the following terms with suitable illustrations- 10

(i) Null hypothesis

(ii) alternative hypothesis

(iii) Critical region

(iv) Type I Error and Type II Error

(v) Level of significance.

(b) A large population has a mean height of 150 cm. and a standard deviation of 20 cm. A random sample of size 100 is taken from this population. Find the probability that the sample mean will (i) exceed 151 cm. (ii) lie between 148 cm and 155 cm. Assume that the population of heights is normal and that the sampling is with replacement. 10

(c) A certain analysis reveals that the duration (x, in seconds) of pauses that occur in a monologue follows exponential distribution with p.d.f. 10

$$f(x,\theta) = \frac{1}{\theta} e^{-x/\theta} \qquad x > 0,\ \theta > 0$$

$$= 0 \qquad \text{otherwise.}$$

A certain recorded monologue is considered for testing $H_0 : \theta = 0.7$ against $H_1: \theta = 1$. It is decided to measure the duration of a randomly chosen pause from the monologue then reject H_0 if the duration of this pause is at least 1.5 seconds. Find the probabilities of both types of error.

(d) A group of 121 boys obtained mean intelligence quotient (I.Q.) of 84 while a group of 81 girls obtained 80. If the s.d. of I.Q. is given to be 10, can we conclude that there is a significant difference between their performances? Use 5% level of significance. 10

Q.4. (a) State the probability density function of a Normal distribution with mean μ and variance σ^2. State the important properties of this distribution. 5

(b) Explain how would you arrive at the best decision criterion based on a large sample to test the hypothesis $H_0 : \mu = \mu_0$ against $H_1 : \mu > \mu_0$ where μ_0 is specified constant and μ is the mean of a population under consideration, if you use 5% level of significance. 5

(c) Find the cumulative distribution function F(x) for the random variable X with p.d.f. as 5

$$f(x) = x \qquad 0 < x < 1$$

$$= 2 - x \qquad 1 \le x < 2$$

$$= 0 \qquad \text{otherwise}$$

Find mean and variance.

(d) An item is packed in lots of 100 each. Let m denote the mean number of defectives in a packet of 100. To test $H_0 : m = 2$ against $H_1 : m = 3$, it is decided to select one packet and inspect the items in it. If it contains four or more defectives, it is proposed to reject H_0. Find the level of significance for the test. 5

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